Projectile Motion

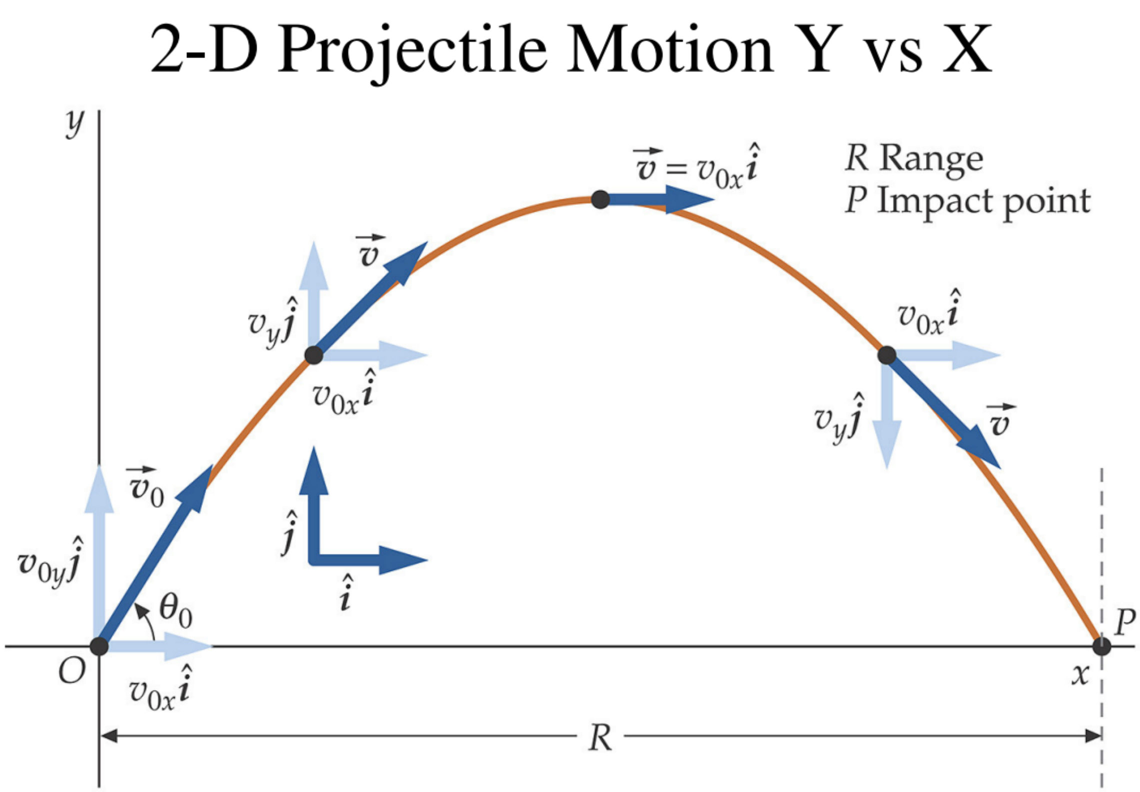
The motion of an object in a vertical plane under the effect of gravitational force is known as "**projectile motion**".

(ان حركة جسم في مستوى عمودي تحت تأثير قوة الجاذبية يعرف بحركة المقذوفات)

The approach to 2-D projectile problems is to resolve the velocity vector into **horizontal** and **vertical components.**

(تعتبر حركة المقذوفات من الامثلة للحركة في بعدين ، سوف نقوم بايجاد معادلات الحركة للمقذوفات لتحديد الازاحة الافقية والراسية والسرعة والتعجيل)

- The vertical component is affected by gravity.
- The horizontal component is unchanged



Projectile Motion: Horizontal Motion الحركة الافقية

- $a_x = 0$ لان القذيفة تتحرك متأثرة بالتعجيل الارضي والذي يتجه شاقوليا نحو الاسفل)
- The velocity in horizontal direction remains constant.

$$v_x = v_o \cos\theta$$

- The position in x direction can be determined by

$$x - x_o = v_{ox}\, t$$

$$x - x_o = v_o \cos\theta\, t$$

Projectile Motion: Vertical Motion(الحركة العمودية السقوط الحر)

- $a_y = -g$ The velocity along the y-axis
- The velocity in Vertical direction remains constant.

$$v_y = v_o sin\theta - gt$$

- The position in x direction can be determined by

$$y - y_o = v_{oy}\, t - \frac{1}{2}\, g\, t^2$$

$$y - y_o = v_o \sin\theta\, t - \frac{1}{2} g\, t^2$$

Horizontal range and maximum height of a projectile

It is very important to work out the range (R) and the maximum height (H) of the projectile motion.

Horizontal Range: The distance OA is defined as the horizantal range R

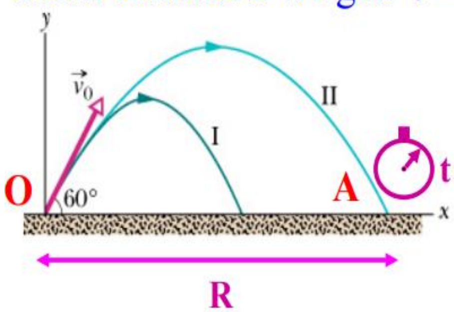
At point A we have: $y=0$ From equation 4 we have:

$(v_0 \sin\theta_0)t - \frac{gt^2}{2} = 0 \rightarrow t\left(v_0 \sin\theta_0 - \frac{gt}{2}\right) = 0$ This equation has two solutions

Solution 1. $t=0$ This solution correspond to point O and is of no interest

Solution 2. $v_0 \sin\theta_0 - \frac{gt}{2} = 0$ This solution correspond to point A

From solution 2 we get: $t = \frac{2v_0 \sin\theta_0}{g}$ If we substitute t in eqs.2 we get:



$\boxed{2\sin A \cos A = \sin 2A}$

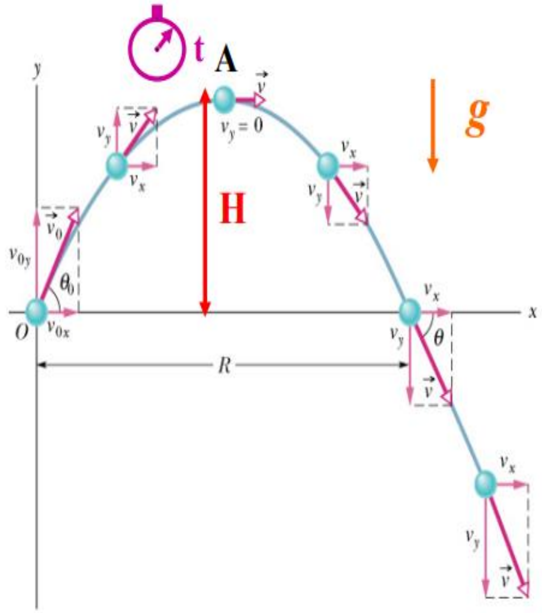
$$R = \frac{2v_o^2}{g}\sin\theta_o \cos\theta_o = \frac{v_o^2}{g}\sin 2\theta_o$$

R has its maximum value when $\theta_o = 45°$

$$R_{max} = \frac{v_o^2}{g}$$

$$R = \frac{2v_0^2 sin\theta cos\theta}{g} = \frac{v_0^2 sin2\theta}{g}$$

Where $sin2\theta = 2\ sin\theta\ cos\theta$



Maximum height H

$$\boxed{H = \frac{v_o^2 \sin^2 \theta_o}{2g}}$$

The y-component of the projectile velocity is: $v_y = v_0 \sin\theta_0 - gt$

At point A: $v_y = 0 \quad \rightarrow v_0 \sin\theta_0 - gt \rightarrow t = \dfrac{v_0 \sin\theta_0}{g}$

$$H = y(t) = (v_0 \sin\theta_0)t - \frac{gt^2}{2} = (v_0 \sin\theta_0)\frac{v_0 \sin\theta_0}{g} - \frac{g}{2}\left(\frac{v_0 \sin\theta_0}{g}\right)^2 \rightarrow$$

$$H = \frac{v_o^2 \sin^2 \theta_o}{2g}$$